INSTABILITATI

Model general

W=energia potentiala in cazul unei deplasari infinitezimale x fata de pozitia de echilibru (cazul unidimensional)

$W(x)=W(0)+x(dW/dx)_0+x^2/2(d^2W/dx^2)_0+\ldots$

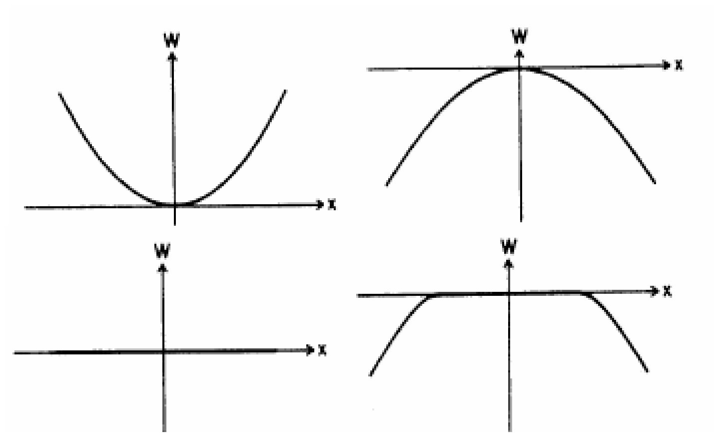
$F=-dW/dx \Rightarrow m\ddot{x} = F = -\frac{dW}{dx}$;

$F_1(x)=-x(d^2W/dx^2)_0 \Rightarrow$ solutia pentru moduri normale : $x=x_0e^{i\omega t} \Rightarrow$

$\boxed{\omega^2=1/m(d^2W/dx^2)_0}$ $\Rightarrow$

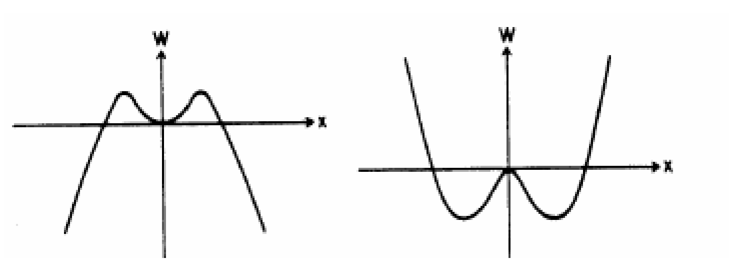
- $(d^2W/dx^2)_0>0 \Rightarrow \omega^2>0 \Rightarrow$ oscil stabila;
- $(d^2W/dx^2)_0<0 \Rightarrow \omega^2<0 \Rightarrow$ oscil instab;
- $(d^2W/dx^2)_0=0 \Rightarrow \omega^2=0 \Rightarrow$ stabilitate neutrala.

Stabilitate liniara



Stabilitate neutrala Metastabilitate

Instabilitati neliniare



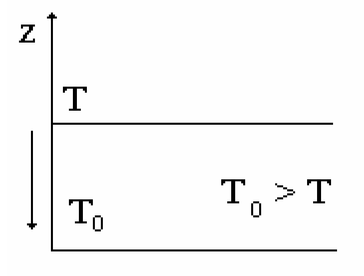
Cu stabilitate liniara Cu stabilitate neliniara

$$\delta W \equiv W(x) - W(0) = \frac{x^2}{2}\left(\frac{d^2 W}{dx^2}\right)_0,$$

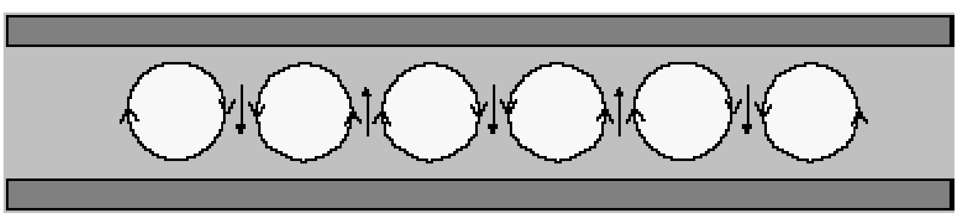
$$\delta W = -\int_0^x F_1(x)\,dx = -\tfrac{1}{2} x F_1(x).$$

O particula este in echilibru stabil daca $\delta W > 0$ pentru orice mica deplasare de la pozitia de echilibru $x=0$.
O particular este intr-o stare de instabilitate daca $\delta W < 0$ pentru cel putin o deplasare elementara, fie ea pozitiva fie ea negative.

Instabilitatea *Rayleigh-Benard*
are loc in fluidele incalzite de jos in sus (supuse unui gradient invers de temperature)

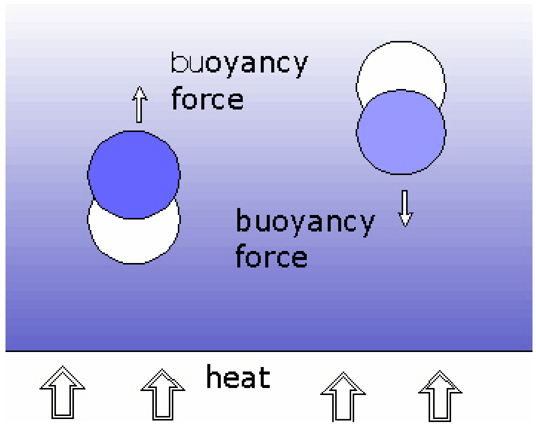


Daca $T > T_0$ atunci sistemul este stabil si prezinta o stratificare in temperature.
Daca $T < T_0$ si daca se introduce o perturbatie in system, atunci pentru o anumita valoare critica a diferentei dintre temparaturi $dT_C = T_0 - T$ pot apare miscari in interiorul fluidului, sistemul devine instabil iar miscarea este organizata in cilindrii care se rotesc in sens invers. Acesti cilindrii se numesc celule *Rayleigh-Benard* si apar atunci cand exista un cuplaj itre campurile dinamice si termice din fluid.



Celule Rayleigh-Benard

In esenta mecanismul instabilitatii Rayleigh-Benard consta in faptul ca in urma procesului de incalzire, are loc o scadere a densitatii in stratul inferior de fluid, strat care incepe sa urce sub actiunea fortei Arhimedice, in timp ce stratul superior cu densitate mult mai mare (temperature mai scazuta) se va prabusi peste stratul inferior.



Atentie, forta Arhimedica trebuie sa fie mai mare decat forta datorata vascozitatii si decat difuzia termica, pentru ca, procesul convectiv sa inceapa in fluid.Efectiv, instabilitatea incepe la anumite valori ale numarului Rayleigh:

$$Ra = \frac{\alpha \Delta T g d^3}{\nu a}$$

Aproximatia Boussinesq

$$d\vec{iv}\vec{U} = 0$$

$$\frac{d\vec{U}}{dt} = -gr\vec{a}d\Pi + \alpha g \theta \vec{z} + \nu \Delta \vec{U}$$

$$\frac{d\theta}{dt} = -\Gamma w + \kappa \Delta \theta$$

unde

$$\Pi = \frac{P}{\rho_0} + gz - \alpha g (T_1 - T_2) z - \frac{1}{2} \alpha g \Gamma z^2$$

$$\Gamma = \frac{T_1 - T_2}{d} = \frac{q}{\kappa}$$

$$\kappa = \frac{k}{\rho_0 C_v}$$

$$\nu = \frac{\mu_n}{\rho_0}$$

$$T(\vec{x}, t) = T_c(z) + \theta(\vec{x}, t)$$

Conditiile la limita pentru viteza: $u = v = w = 0$ pentru z=0 si z=d. Consideram fie temperature constante la limite fie gradienti termici constanti.. Introducem urmatoarele marimi adimensionale:

$[L] = d$ $[\tau] = \frac{d^2}{\nu}$ $[\theta] = d\Gamma$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = -\frac{\partial \Pi}{\partial x} + Pr\Delta u$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z} = -\frac{\partial \Pi}{\partial z} + RaPr\theta + Pr\Delta w$$

$$\frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial x} + w\frac{\partial \theta}{\partial z} = -w + \Delta\theta$$

Conditiile la limita pentru viteza sunt: $u = w = 0$ pentru z=0 si z=1.

<u>Determinarea numarului critic Rayleigh:</u>

Fie:

$$u = -\frac{\partial \psi}{\partial z} \quad v = \frac{\partial \psi}{\partial x}$$

si dupa liniarizare:

$$\frac{\partial}{\partial t}\Delta\psi = -RaPr\frac{\partial \theta}{\partial x} + Pr\Delta^2\psi$$

$$\frac{\partial \theta}{\partial t} = -\frac{\partial \psi}{\partial x} + \Delta\theta$$

Cautam solutii de forma:

$$[\psi(x,z,t), \theta(x,z,t)] = [\Psi(z), \Theta(z)]e^{ik_1 x + \lambda t}$$

ceeea ce inseamna ca sistemul de ecuatii devine :

$$\lambda(D^2 - k_1^2)\Psi = -ik_1 Ra Pr \Theta + Pr(D^2 - k_1^2)\Psi$$

$$\lambda\Theta = -ik_1\Psi + (D^2 - k_1^2)\Theta$$

$$D = \frac{d}{dz}$$

Iar conditiile la limita (z=0, z=1) :

$$\Theta = 0$$

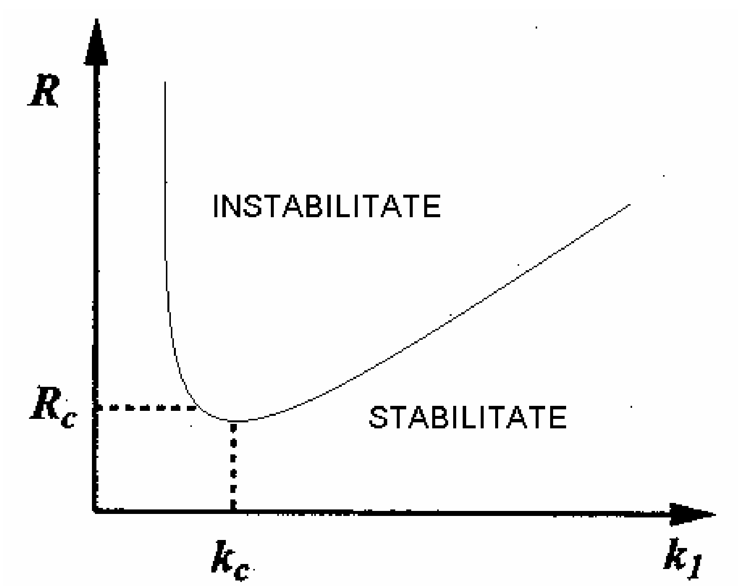
$$\Psi = 0$$

$$D\Psi = 0$$

Rezolvand sistemul se poate determina Valoarea numarului Rayleigh critic :

$$Ra_c = 1707,762$$

$$k_c = 3,117$$



De mentionat ca valorile obtinute sunt specifice unor limite rigide. Pentru cazul in care limita superioara est libera iar cea inferioara este rigida (u=v=w =0 la z=0 si $\frac{\partial u}{\partial z} = \frac{\partial v}{\partial z} = 0$ si w=0 la z=d) valorile sunt:

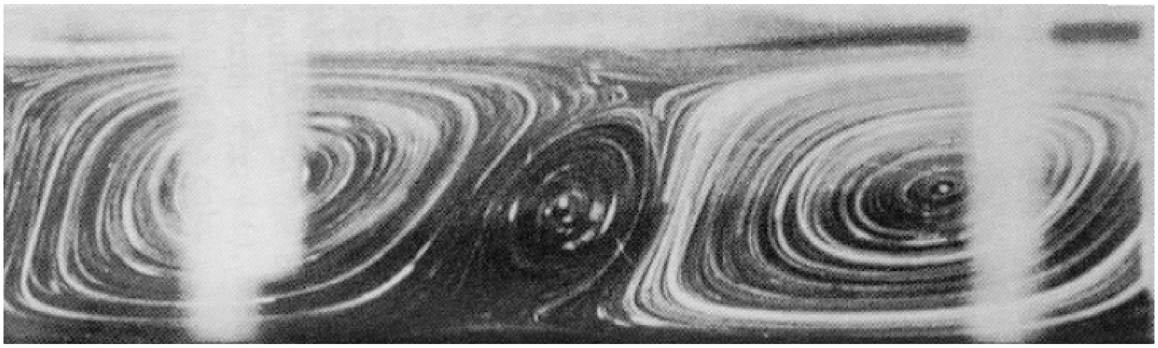
$$Ra_c = 1100,65$$

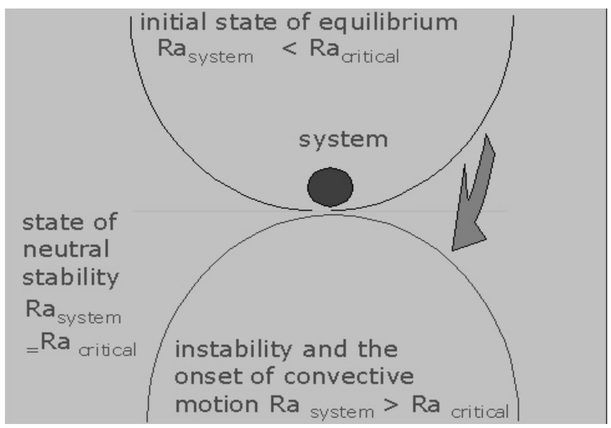
$$k_c = 2,682$$

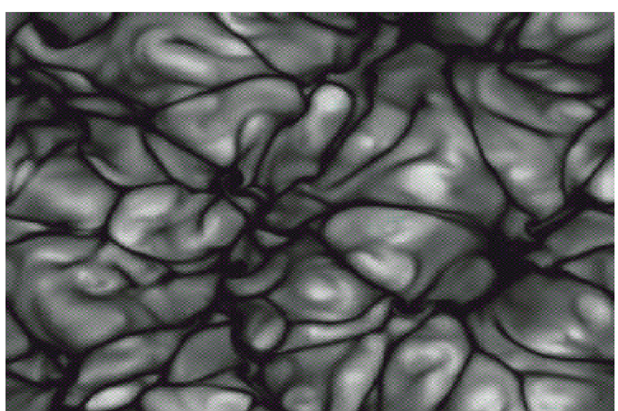
Pentru cazul unor limite libere ($\frac{\partial u}{\partial z} = \frac{\partial v}{\partial z} = 0$ si w=0 la z=0 si z=d) valorile devin:

$$Ra_c = 657,511$$

$$k_c = 2,2214$$







Convectia la suprafata Soarelui cu Ra=5 10^5 (obs Cattenao 2001)